A rationale for the eight aspects in the mathematics framework

How the aspects were selected

The mathematics framework provides a high-level map to describe what progress in mathematics and statistics looks like. It illustrates this progress by showing how students respond when problems become more complex and students need to apply their skills and knowledge in more complex ways. This focus on solving problems does not negate the importance of students learning mathematical facts and procedures. It does, however, emphasise the pointlessness of knowledge without understanding and without the ability to apply that knowledge.

The Ministry of Education convened workshops with mathematics and statistics experts (academics, researchers, and teachers) to discuss the ways in which mathematics should be organised for the Learning Progression Frameworks. The experts advised that the mathematics framework should be structured to further support the effective teaching of the mathematics and statistics learning area of the New Zealand Curriculum (NZC). They also advised that the framework should build on the research and experience of the Numeracy Development Projects. The proposed aspects for the mathematics framework were organised according to the strands of the mathematics and statistics learning area in order to ensure that the aspects:

- reflected the key ideas of the learning area;
- reflected the breadth and emphasis of the learning area;
- aligned with mathematics programmes in schools and the ways in which mathematics is taught and assessed;
- would be readily understood by teachers.

The eight aspects of the mathematics framework cover the breadth of the mathematics and statistics learning area and emphasise making sense of mathematical ideas and reasoning mathematically. Four of the aspects address key ideas in the number and algebra strand, two address the geometry and measurement strand, and two address the statistics strand (see Table 1).

Table 1: Mathematics aspects: initial and final aspects

Strands	Initially proposed aspects	Final aspects
Number and algebra	Additive thinking	Additive thinking
	Multiplicative thinking	Multiplicative thinking
	Proportional reasoning	Patterns and relationships
	Algebraic thinking	Using symbols and expressions to think mathematically
Geometry and measurement	Measurement sense	Measurement sense
	Geometric thinking	Geometric thinking
Statistics	Statistical thinking	Statistical investigations
	Statistical literacy	Interpreting statistical and chance situations

Following the initial workshops, the aspects were developed by mathematics educators according to their subject-matter expertise. As a result of developing the signposts for each of the aspects, the subject-matter experts proposed changes to four of the initially proposed aspects.

The two algebra-related aspects, proportional reasoning and algebraic thinking, were further considered by a researcher who had undertaken a significant piece of work developing an algebraic framework[1]. This work pointed clearly to the fundamental importance of the patterns and relationships aspect, but also made evident the significance of using symbols and expressions to think mathematically across the whole learning area. This led to the final two algebra-related aspects shown in Table 1. It also resulted in proportional reasoning being omitted as a separate aspect in the framework. The rationale for this was that the Number Framework unnaturally separated proportional reasoning from the two pairs of reciprocal number operations, and in particular from multiplication and division. The decision was made to embed proportional reasoning within the multiplicative thinking aspect and the two algebra-related aspects.

The two statistics aspects, statistical thinking and statistical literacy, were reviewed by a small team with expertise in this area. Their work made clear the distinction between students' enactment of the statistical investigations cycle, and students' ability to interpret their own and others' data as well as chance situations. This led to the final two statistics aspects shown in Table 1.

The development of the measurement sense aspect built on established learning progressions that start from identifying the attribute (for example, length or weight) and progress through the use of non-standard measurement units to the use of standard measurement units and formulae.

The geometric thinking aspect was challenging because the progression needed to describe increasing sophistication in three related but different elements of geometry: shape, transformation, and position and orientation. This meant that the aspect, through the sets of illustrations, needed to describe developing expertise in each of the three elements. In addition, less is known about the development of geometric expertise in primary education. For example, the most cited framework of geometric thinking (van Hiele[2]) describes five levels of thinking, but only two of these are applicable to levels one to four of the NZC. These two factors resulted in the identification of fewer signposts than the other aspects.

The eight aspects in the mathematics framework

The mathematics framework comprises eight aspects that reflect the breadth and emphasis of the mathematics and statistics learning area of the NZC.

Additive thinking

This progression is similar to the additive domain of the Number Framework in that it focuses on the increasingly sophisticated and flexible addition and subtraction strategies that students develop to solve increasingly complex problems. However, the sets of exemplars are not a direct match to the stages of the Number Framework. For example, imaging (stage 3 in the additive domain) is not identified by a discrete set of exemplars, and the higher stages of the domain are represented by more than one set of exemplars.

Multiplicative thinking

This progression combines elements from both the multiplicative and proportional domains of the Number Framework. However, as with additive thinking, the sets of exemplars are not a direct match to the stages of the Number Framework. This progression focuses on students' ability to think multiplicatively as they solve multiplication, division, and proportional problems involving an extended range of whole numbers, decimals, fractions, ratios, and percentages, and in a range of contexts.

Patterns and relationships

This algebraic thinking progression develops students' understanding of the structure of and the relationships between numbers, shapes, and measures. In exploring patterns of increasing complexity, students develop the ability to recognise, explain, and generalise relationships between quantities and objects.

Using symbols and expressions to think mathematically

This algebraic thinking progression is fundamental to all other aspects of mathematics. It focuses on the ways in which symbols, expressions, and equations are used to communicate mathematical ideas. In solving problems in a range of contexts, students must make sense of the symbols they read and must be able to express their understanding of a problem using the symbolic language of mathematics.

Geometric thinking

As students make sense of and navigate their spatial world, they come to recognise, describe, and use the properties and symmetries of shapes, and they describe movement and position with increasing accuracy.

Measurement sense

Understanding what a measurable attribute is and becoming familiar with the units, systems, and processes that are used in measuring attributes is the focus of this progression. Progression in the understanding of measurement is determined by increased sophistication in the measurable attributes of objects and the complexity of the attribute being measured.

Statistical investigations

The statistical investigations progression is based on the students' development of an increasingly sophisticated implementation of the statistical inquiry cycle that includes posing investigative questions, collecting data, displaying data, and discussing results.

Interpreting statistical and chance situations

As students are exposed to the statistical evidence presented by others, they need to be able to interpret and gain information from what they see, critically evaluating both the quality of the evidence and the arguments being presented on the basis of that evidence.

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[1] Linsell, C., & Tozer, L. (2010). Algebra: More than just patterns. In R. Averill & R. Harvey (Eds.), *Teaching primary school mathematics and statistics: Evidence-based practice*. (pp. 75-87). Wellington, New Zealand: NZCER Press.

[2] Van Hiele, P. M. (1986). Structure and Insight. Orlando: Academic Press.